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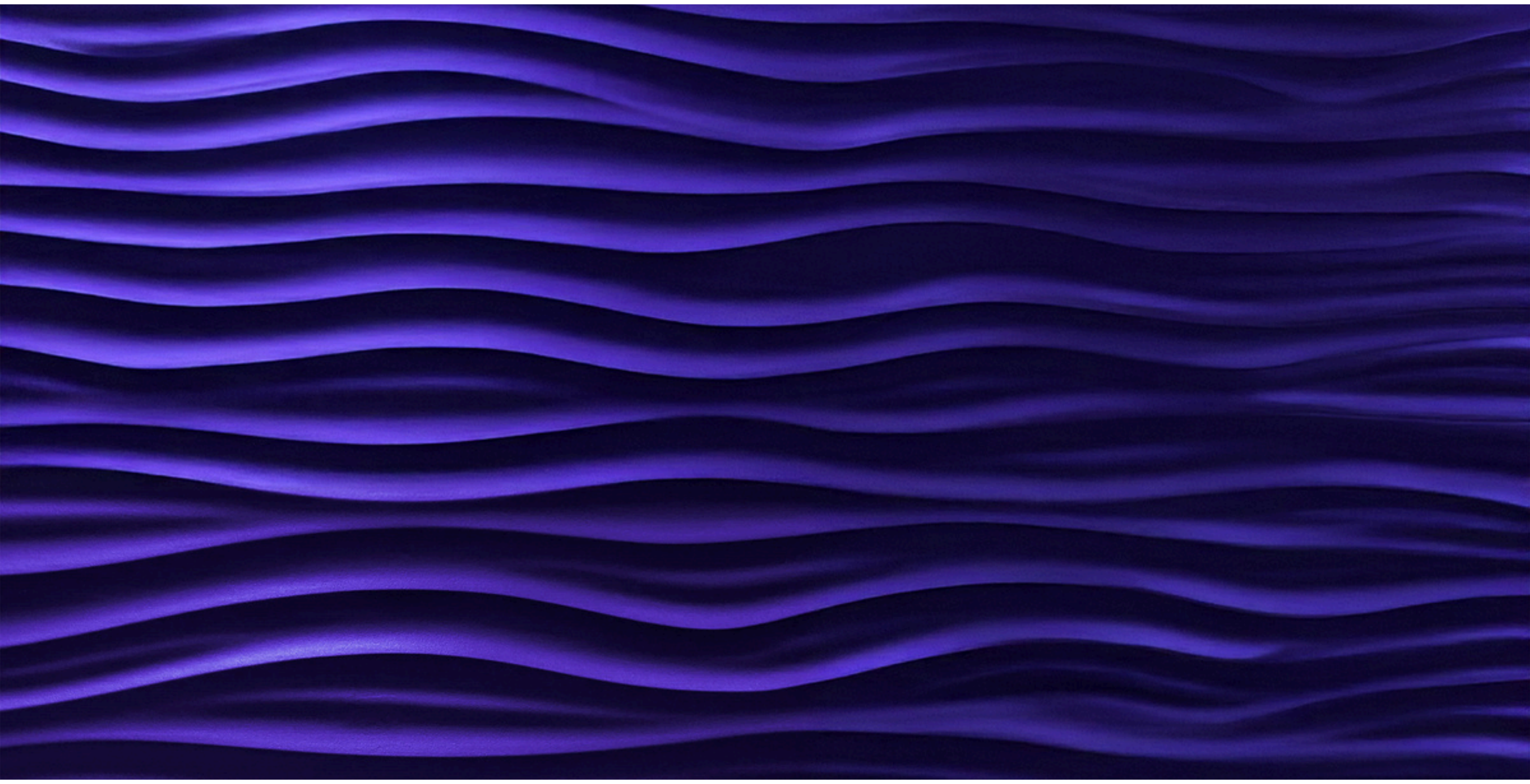

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Analyzing nonlinear systems in the frequency domain with harmonic balance in Quanscient Allsolve

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Abstract

Nonlinear systems appear frequently in engineering and pose unique challenges in simulation due to their complex, often non-intuitive behavior. Traditional time-domain approaches such as transient simulations can be slow, require extensive tuning, and may fail to capture essential frequency-domain characteristics. As demand grows for faster and clearer insights into nonlinear behavior, more efficient and scalable alternatives are needed. The harmonic balance method offers a frequency-domain approach that resolves nonlinear periodic steady-state responses by decomposing system behavior into fundamental and higher-order harmonics.

This paper presents a frequency-domain simulation strategy for nonlinear systems using harmonic balance in Quanscient Allsolve, a cloud-native multiphysics simulation platform. The method is applied to engineering cases involving electrical, structural, and fluidic domains, ranging from Joule heating and nonlinear beam vibration to microspeaker behavior, each demonstrating key benefits of harmonic analysis.

The workflow integrates finite element analysis (FEA) with harmonic decomposition and runs efficiently at scale using cloud-based parallel computing. These capabilities allow engineers to simulate large, coupled systems while selectively analyzing dominant harmonics, reducing computational time and improving clarity in the results.

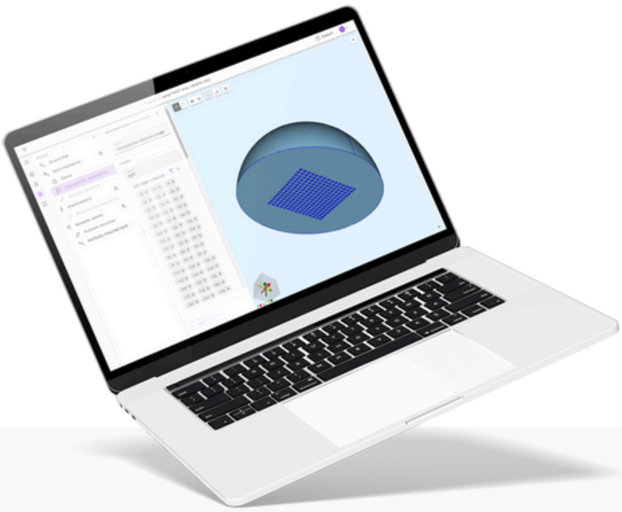
The findings illustrate the advantages of harmonic balance for analyzing nonlinear phenomena and demonstrate how scalable, cloud-native tools enable practical use of this method in real-world engineering problems.

Keywords — Harmonic balance; nonlinear simulation; frequency domain; Finite Element Method (FEM); cloud computing; multiphysics analysis; periodic steady-state.

Introduction to Quanscient Allsolve

The cloud-based multiphysics simulation platform Quanscient Allsolve was used for all simulations featured in these case studies.

Learn more at quanscient.com ⟶



Quanscient Allsolve

- A cloud-based FEM platform for fast and scalable multiphysics simulations
- Developed by Quanscient, founded in 2021 in Tampere, Finland
- Enables fully coupled multiphysics simulations across all core physics domains

Trusted in both industry and academia

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The harmonic balance method

Many engineered systems involve periodic or oscillatory behavior, such as vibrating components or alternating electrical signals. Accurately understanding their steady-state response is key to ensuring reliable performance. FEA is often used for this but can be challenging when the system includes complex shapes or coupled physics.

Traditional time-domain simulations simulate over time until the system settles to it's steady-state operation, which can be slow and computationally heavy. In addition, extracting detailed frequency information from these simulations isn't always straightforward, and the extracted spectrum can be noisy, depending on the time step.

The harmonic balance method offers an alternative by working directly in the frequency domain. It solves for steady-state responses as a combination of harmonics, making the analysis more efficient and easier to interpret.

Challenges in simulating steady-state periodic behavior

Long runtimes: Time-domain simulations need many cycles to reach steady state, resulting in high computational costs.

Frequency extraction issues: Extracting clear harmonics from transient data often requires extra processing that can introduce noise.

High resource demands: Large FEM models with many degrees of freedom require significant memory and processing power.

Traditional harmonic balance limits: Applying harmonic balance to large FEM models has been restricted by computational challenges.

Multiphysics complexity: Coupled physical domains increase simulation complexity, making traditional methods less efficient.

Quanscient Allsolve's solution

Quanscient Allsolve leverages on-demand cloud computing resources to overcome traditional hardware limitations in terms of memory and processing power.

The strongly coupled formulations in Allsolve robustly enable complex multiphysics simulations with harmonic balance method at an unprecedented large scale, provides engineers with clear insights into complex periodic behaviors.

The simulation examples presented here demonstrate how Quanscient Allsolve applies harmonic balance to a representative engineering problem, highlighting improved efficiency and accuracy over conventional methods.

Case example
AC Joule Heating [3]

Simulation objective

Electrical systems, from the wiring in our homes to the circuits in our smartphones, generate heat as electricity flows through their conductors. This phenomenon, known as Joule heating, can significantly impact the performance and lifespan of these systems.

Simulations are used to predict and analyze Joule heating effects. By simulating different geometries, materials, and current loads, efficiency can be optimized in wire designs. This is especially important in applications with high currents or limited space, such as electronic devices and power transmission systems.

A simple example was presented to illustrate the key principle of the harmonic balance method.

Simulation model

The model was an aluminium filament with a rectangular cross section of 0.1 mm × 0.1 mm and a length of 1 mm (see Fig. 1).

It included current flow and heat solid physics, coupled through Joule heating.
The filament was driven on one end by a sinusoidal current at a chosen fundamental frequency (f_0), with the other end grounded.

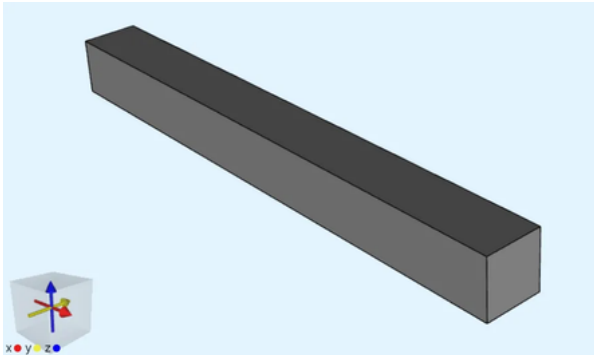

Fig.1: Model view of the aluminium filament in Quanscient Allsolve.

Results and discussion

Plots in Fig. 2 show the evolution of temperature for the current flow through the filament at fundamental frequency f_0 = 100 and 400 Hz.

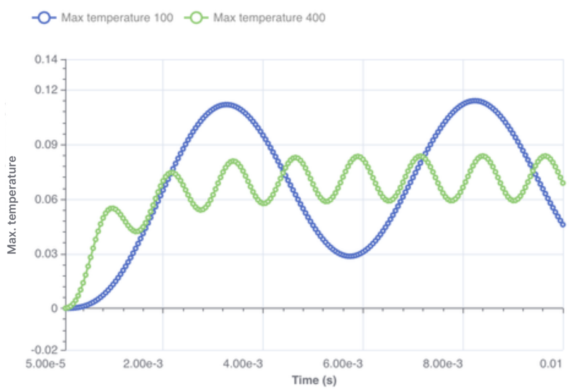

Fig. 2 Temperature in transient simulations at f_0 = 100 Hz and 400 Hz.

Case example
AC Joule Heating

The maximum temperature plot at steady state consists of a constant component and a periodically fluctuating component at a frequency twice that of the fundamental frequency ($2 \times f_0$).

These components can be effectively captured using the harmonic balance method in the frequency domain, as illustrated in Fig. 3.

Here, T_1 represents the constant component at zero frequency; T_2 and T_3 correspond to the sinusoidal and cosinusoidal parts at the fundamental frequency, respectively; while T_4 and T_5 denote the sinusoidal and cosinusoidal parts at twice the fundamental frequency.

Joule heating is proportional to the square of the applied current (I^2), so the sinusoidal alternating current at the fundamental frequency causes constant heating of the filament with temperature fluctuations around this steady level.

$$\dot{Q} \propto I(t)^2$$

$$\dot{Q} \propto I_0^2 \sin^2(2\pi f0\, t)$$

$$\dot{Q} \propto I_0^2 \frac{1}{2}(1 - \cos(2\pi\, 2f0\, t))$$

The temperature is governed by the diffusion equation.

$$\rho C_p \dot{T} = k\nabla^2 T + \dot{Q}$$

The transient term adds damping to the system with the temperature field, resulting in both sinusoidal and cosinusoidal components.

$$T \propto \frac{1}{2} I_0^2(\vec{x}) + \phi_{s1}(\vec{x}) \sin(2\pi\, 2f0\, t) + \phi_{c1}(\vec{x}) \cos(2\pi\, 2f0\, t))$$

As clearly visible, in addition to the constant component, the rest of the contributions come from the second harmonic coefficients at frequency 2 x f_0, and the fundamental harmonics (at f_0) are zero.

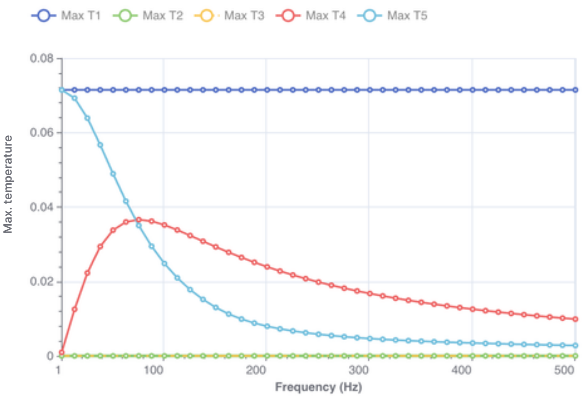

Fig. 3: Harmonic components of temperature in harmonic balance simulations at f_0 between 1 Hz to 501 Hz in 41 steps.

Advantages of the method

The comparison with the transient simulation approach clearly demonstrated the key benefits of the harmonic balance method.

It highlighted the method's ability to efficiently capture steady-state behavior and nonlinear responses in the frequency domain, offering a more targeted and computationally efficient alternative to traditional time-domain simulations.

Case example
Backbone curves [4]

Simulation objective

Beams are fundamental structural elements used in a wide range of applications, from bridges and aircraft to microelectronics. However, beams are susceptible to vibrations, which can lead to fatigue, instability, and potential failure. Therefore, understanding beam vibration behavior is essential for engineers across various disciplines.

In this paper, a demonstration of the harmonic balance method's capability to capture the nonlinear behavior of the clamped-clamped beam is presented.

Simulation model

The simulation model consists of a beam with a rectangular cross section measuring 0.03 m × 0.03 m and a length of 1 m.

The analysis includes solid mechanics and mesh deformation physics, with the coupling accounting for large deformation effects due to geometric nonlinearity.

The beam is driven by a sinusoidal load applied at a selected fundamental frequency (f_0).

Results and discussion

Mechanical resonance is represented by plotting the maximum displacement as a function of the driving frequency.

Fig. 4 shows the maximum displacement across frequencies in the absence of geometric nonlinearity. The response exhibits linear behavior, with a clear resonance peak visible.

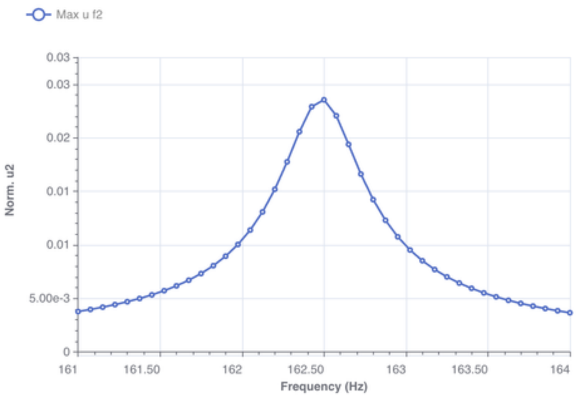

Fig. 4: Maximum displacement vs. frequency sweep without geometric nonlinearity.

Case example
Backbone curves

When geometric nonlinearity is taken into account, multiple valid solutions can exist at a given driving frequency, as supported by findings in the literature [5] (see Fig. 5). This behavior, illustrated by the characteristic backbone-shaped curve, highlights the challenges of using transient simulations for such cases.

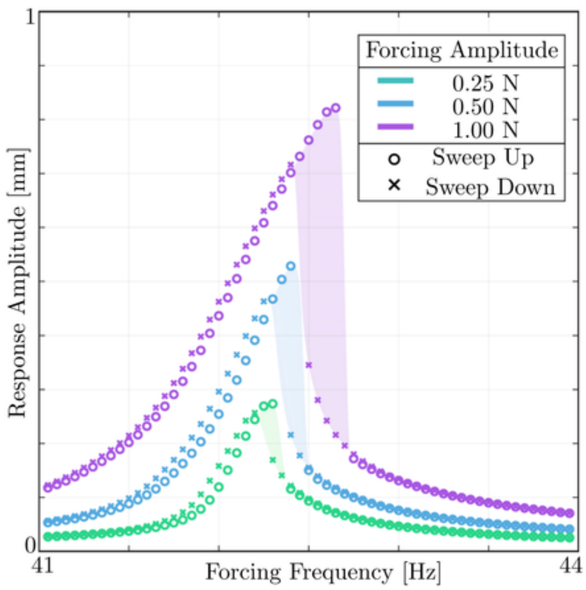

Fig. 5: Experimental data from Hayashi et al. [5].

However, Quanscient Allsolve is able to trace the points on each branch using the harmonic balance approach, as shown in Fig. 6.

It is important to note that the simulation setup in this example differs from that of Hayashi et al. [5] and is therefore not directly comparable, nor is it intended to be.

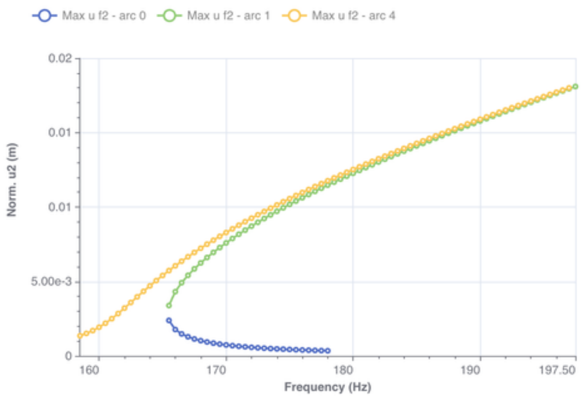

Fig. 6: Maximum displacement vs. frequency sweep with geometric nonlinearity showing three different branches.

Advantages of the method

The ease of simulating nonlinear responses and the demonstrated advantage of using the harmonic balance method over transient simulations are shown using Quanscient Allsolve.

Case example
Microspeaker

Simulation objective

Electrostatically actuated silicon-based microspeakers are an emerging technology due to their obvious advantages, providing high sound quality over a wide range of frequencies [8, 9].

Showcasing more advanced multiphysics capabilities including mesh deformation on a complex application case of electrostatically actuated silicon-based microspeakers

Simulation model

The system consists of two parallel plates separated by an air gap, involving physics domains, such as electrostatics, solid mechanics, laminar flow, and mesh deformation.

Key couplings include the electric force interaction between electrostatics and solid mechanics, as well as fluid-structure interaction between solid mechanics and laminar flow.

Large displacement effects introduce geometric nonlinearity coupling electrostatics, solid mechanics, and laminar flow with mesh deformation through an arbitrary Lagrangian-Eulerian formulation.

The plates are actuated electrostatically by an applied sinusoidal voltage at a chosen fundamental frequency (f_0), and the first three harmonics of this excitation are considered in the analysis.

Results and discussion

The simulation was performed at a driving frequency of 100 Hz, with 800,000 degrees of freedom, using 12 processing cores, and a runtime of 30 minutes.

Fig. 7 presents qualitative results through contour plots of displacement in the solid region, which consists of the parallel plates, and fluid velocity magnitude in the fluid region sandwiched between the plates.

Case example
Microspeaker

A cross-section through the center, shown in Fig. 8, provides additional information with velocity vectors.

It is important to note that the fields depicted in this example represent a time-dependent reconstruction over one full time period based on the harmonic data.

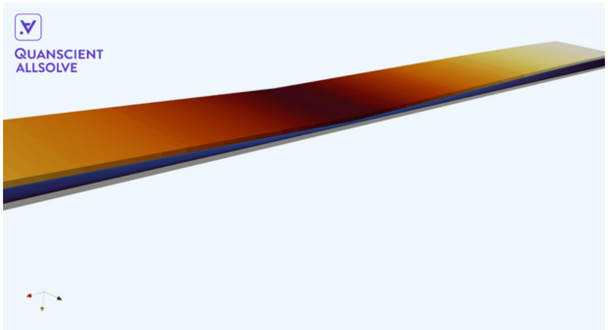

Fig. 7: Time-dependent displacement field in the solid region (parallel plates) and the fluid velocity magnitude in the fluid region (sandwiched between parallel plates).

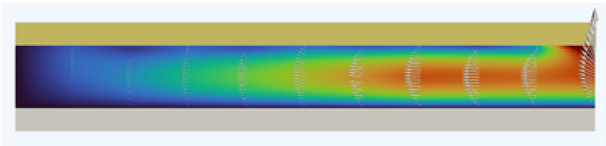

Fig. 8: Cross-section at the center with time-dependent displacement field in the solid region (parallel plates) and the fluid velocity field along with vectors in the fluid region (sandwiched between parallel plates).

The quantitative comparison of the normalized displacement of the top microbeam along its length on the centerline is shown in Fig. 9, alongside normalized measured data from the published article [9], validating the simulation approach.

The observed discrepancies may be attributed to the relatively coarse mesh used in this example.

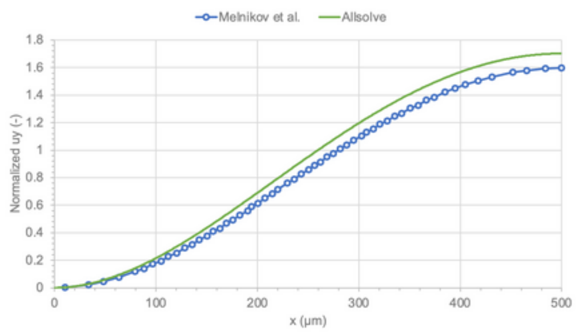

Fig. 9: Comparison of spatial displacement field (reconstructed from the harmonic data) at the center of the beam.

Case example
Microspeaker

A simultaneous sweep over frequency and voltage was performed with 75 simulations covering frequencies from 20 Hz to 20 kHz and voltages from 5 to 25 V, using 416,000 degrees of freedom.

Each simulation ran for 12 minutes on 8 cores. Fig. 10 illustrates the total harmonic distortion as a function of driving frequency and voltage, a common metric to quantify sound signal quality.

The harmonic balance approach makes it straightforward to conduct such parameter sweeps and explore the design space to identify optimal designs.

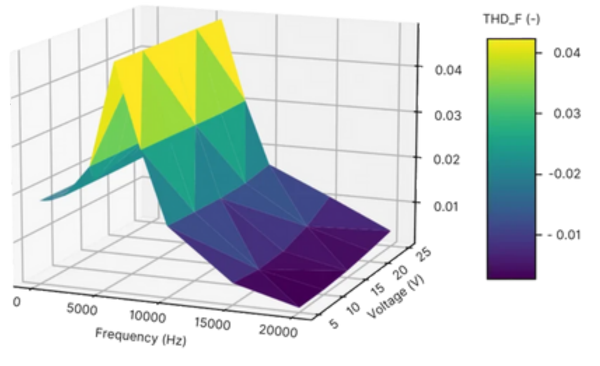

Fig. 10: The total harmonic distortion as a function of driving frequency and voltage.

Advantages of the method

The harmonic balance method was applied to a fully coupled multiphysics problem involving electrostatics, solid mechanics, and fluid dynamics, including mesh deformation, geometric nonlinearity, and prestress.

The nonlinear behavior was clearly captured and validated against literature data.

Frequency and voltage sweeps offered valuable insights into their effects on the output within the design space, aiding in the selection of an optimal design.

Computational advantages were demonstrated through the efficient execution of parallel sweeps.

Conclusion

1

Nonlinear systems are central to many engineering applications, and analyzing their behavior accurately is essential for designing reliable and efficient devices. The **harmonic balance method** provides a robust alternative to traditional time-domain simulations by capturing the full steady-state response directly in the frequency domain, offering both computational efficiency and greater clarity in interpreting system dynamics.

2

By addressing the typical computational challenges associated with large-scale, fully coupled multiphysics models, including those involving geometric nonlinearity and mesh deformation, through cloud-based infrastructure, **Quanscient Allsolve** enables engineers to apply the harmonic balance method to highly detailed simulations that would be impractical or too time-consuming to solve with conventional tools.

3

The application of this approach to cases, such as **Joule heating, vibrating beams, and microspeaker designs,** demonstrates its versatility and its ability to deliver engineering-relevant output, such as harmonic distortion or displacement profiles, more quickly and accurately, making it a practical tool for design exploration and decision-making in complex systems.

Key takeaways

- Nonlinear systems are common in real-world applications and are often hard to analyze with traditional transient methods, which can be slow and noisy
- The harmonic balance method provides a faster, more accurate way to analyze these systems by breaking their behavior into simple, periodic patterns, including those caused by system complexity
- Cloud-based multiphysics simulation software Quanscient Allsolve makes this method practical for detailed simulations, handling large and complex models with the help of cloud computing
- Real-world examples, such as thermal analysis of wires, beam vibrations, and microspeaker designs, highlight the method's usefulness and efficiency

Get in touch

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